G.C.E A/L Support Revision Programme -2013

Revision Paper | Combined Mathematics -1

Ministry of Education

this Paper consists of two parts

Part A consists of 10 questions Answer all questions

Part B consists of 7 questions answer any 5 questions

Time – Three Hours

Part A

1) Sketch the graphs $y = |x+1|$ and $y = |2x+1|$ in the same figure. Find the range of values of x that satisfy the inequality $|x+1| < |2x+1|$.

2) Consider taking all the letters ECCENTRICTTY. Find the number of permutations that can be formed
   i) With no restrictions what so ever in the order of letters
   ii) With a letter "T" occurring immediate after N.

3) Find the coefficient of x^{2013} in the expansion $(1+x)^{2012}(1-x+x^2)$

4) Evaluate i) $\lim\limits_{x\to 0} \frac{\sqrt{1+x-x^2}-1}{x}$ ii) $\lim_{x\to\infty} \frac{1+x-3x^2}{x^2-4x+8}$

5) Differentiate with respect to x $3^{\left(1/_{x+1}\right)} \quad y = 4$

6) Evaluate $\int_{-2}^{2} |x+1|\, dx$

7) Find the equation of the bisector of the angle between the two straight lines $3x+4y+5=0$ and $12x-5y-3=0$ for the angle which contains the point of origin

8) Find the coordinates of the point P which bisects the two points $A(4,0)$ and $B\ (3,-3)$ externally in the ratio 3:1 .

9) Find the equation of the circle which touches the line 4x – 3y – 6 = 0 and the co-ordinate axes.

10) Deduce that $\tan(x+y) = \frac{\tan x + \tan y}{1-\tan x \tan y}$ using the expansions of $\sin(x+y)$ and $\cos(x+y)$.
    if A+B+C=π show that $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{A}{2}\tan\frac{C}{2} = 1.$

Part B

Answer any 5 questions

11) (a) The graph of $y = x^2 - (k-2)x + k + 1$ intersect the x-axis at two distinct points $(\alpha, 0)$ and $(\beta, 0)$; $k \in \mathbb{R}$

(i) Find the range of values k can take.

(ii) Also if ($|\alpha + \beta| < 5$ Find the range of values k can take.

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(b) Show $\frac{x^3+4x^2-10x+6}{(x-3)(x^2+4)}$ in partial fractions

12) (a)

$u_1, u_2, u_3, \dots, u_n, \dots$ is a positive number sequence. They are in an arithmetic series. Prove that $\frac{1}{u_1u_2} + \frac{1}{u_2u_3} + \frac{1}{u_3u_4} + \dots + \frac{1}{u_{n-1}u_n} = \frac{n-1}{u_1u_n}$ applying the principals of mathematical induction for $n \geq 2$.

(b) Given $v_r = r(r+1)$ and $f(r) = Ar^2 + Br + C$; $r \in \mathbb{Z}^+$. Find the constants A, B and C such that $v_r = f(r) - 3f(r-1)$ hence evaluate $\sum_{r=1}^{n} \frac{v_r}{3^r}$

Is this series convergent? Justify your answer.

13) (a) Given 3 matrices

A=$\begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 \\ 0 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 5 \\ -4 & k \end{pmatrix}$

Find the value of k such that $A(2B - C) = 6\,I$ where I is the unit matrix.

Hence deduce A^{-1} .

(b) (i) $\left(\frac{3-i}{3+i}\right) Z = \frac{5+5i}{-1+2i}$ express the complex number Z in the form a+ib

(ii) ABCD is a rhombus in the argon diagram with the vertices in the anticlockwise sense. There points represent the complex numbers Z_1, Z_2, Z_3 and Z_4 . ABC $\measuredangle = \frac{\pi}{3}$

Show that $Z_2 = \frac{1}{2} Z_1 \left(1 + \sqrt{3i}\right) + \frac{1}{2} Z_3 \left(1 - \sqrt{3i}\right)$

And $Z_4 = \frac{1}{2} Z_1 \left(1 - \sqrt{3i}\right) + \frac{1}{2} Z_3 \left(1 + \sqrt{3i}\right)$.

(iii) R is the region that satisfy $|Z - 4i| \leq 2$ in the argon diagram. Find the difference of the highest volume and the lowest value of $\arg Z$ in this region.

Determine the highest and the lowest values of $|Z|$

14) (a)

(i) $a(x-y) = \sqrt{1-x^2} + \sqrt{1-y^2}$, $a \in \mathbb{R}$ and " a " is a nonzero constant show that $\tan\frac{\theta-\alpha}{2} = \frac{1}{a}$ by substituting $x = \sin\theta$ *and* $y = \sin\alpha$ such that $\theta + \alpha \neq \pi$.

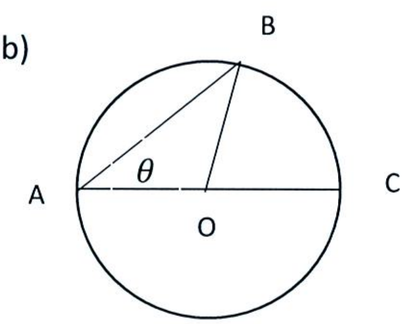
Hence show that $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

(ii) Is $y = \frac{x\sin^{-1}x}{\sqrt{1-x^2}}$ show that

$$(1-x^2)\frac{d^3y}{dx^3} - 5x\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = 0$$

Find $\left(\frac{d^3y}{dx^3}\right)$, x=0

(b)



The figure shows a circular lake with radius 2km and center O. A is a fixed point AB is a variable chord. The constant speed a person can swim along AB $2\sqrt{3}\ kmh^{-1}$. One can walk along the bank from B to C at a constant speed of of $4kmh^{-1}$. $BAO\measuredangle = \theta$ It takes T(θ) hours for the man to go from A to C by swimming at constant speed from A to B and walking at constant speed from B to C.

Find T(θ). Find the θ,maximum time taken to go from A to C by examining the sign of the first derivative.

15) (i) Find $\int_0^{\frac{\pi}{2}} \frac{\cos x}{3+\cos^2 x}dx$ by a suitable substitution.

(ii) Given $I_n = \int \frac{1}{(a+x^2)^n}dx$ for $n \geq 1$.

Show that $I_{n+1} = \frac{x}{2an(a+x^2)^n} + \left(\frac{2n-1}{2an}\right)I_n$

and hence find $\int \frac{1}{(4+x^2)^3}\ dx$.

16) Equation of the side OA of a Rhombus OABC 4x-3y=0 The equations of the diagonal OB is x – y = 0. O is the origin. Find the Equation of OC .If B = (5 , 5) find the equations of AB, AC and BC.

A circle with radius 1 unit is drawn to touch the sides OC and OA of the rhombus OABC.

Find the equation of the circle.

Find the Equation of the tangent chord of the tangents drawn to the circle from O.

17)

I. State the cosine rule, using the usual notations, for a triangle ABC.

BC, CA and AB of a circle ABC are $x + y\,,\ x\,,\ and\ \ x - y$ respectively.

Show that $\cos A = \frac{x-4y}{2(x-y)}$

ii) Show that $(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2 = 4\ cos^2\ \frac{\alpha-\beta}{2}$

Find the general solution of the equation

$(\cos x + \cos 3x)^2 + (\sin x + \sin 3x)^2 = 1$

iii) Solve $2\tan^{-1}(\sin x) - \tan^{-1}(2\sec x) = 0$.