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இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2017 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2017 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2017

සංයුක්ත ගණිතය II
இணைந்த கணிதம் II
Combined Mathematics II

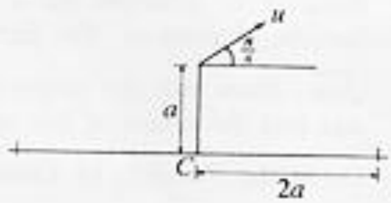
10 E II

Part B

※ Answer **five** questions only.

(In this question paper, g denotes the acceleration due to gravity.)

11. (a) The base of a vertical tower of height a is at the centre C of a circular pond of radius $2a$, on horizontal ground. A small stone is projected from the top of the tower with speed u at an angle $\frac{\pi}{4}$ above the horizontal. (See the figure.) The stone moves freely under gravity and hits the horizontal plane through C at a distance R from C. Show that R is given by the equation $gR^2 - u^2R - u^2a = 0$.



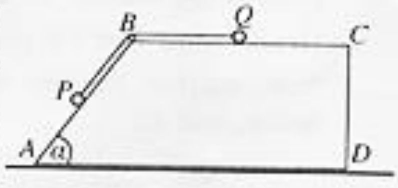
Find R in terms of u, a and g, and deduce that if $u^2 > \frac{4}{3}ga$, then the stone will not fall into the pond.

(b) A ship S is sailing due East with uniform speed u km h^{-1}, relative to earth. At the instant when the ship is at a distance l km at an angle θ South of West from a boat B, the boat travels in a straight line path, intending to intercept the ship, with uniform speed v km h^{-1} relative to earth, where $u \sin\theta < v < u$. Assuming that the ship and the boat maintain their speeds and paths, sketch, in the same diagram, the velocity triangles to determine the two possible paths of the boat relative to earth.

Show that the angle between the two possible directions of motion of the boat relative to earth is $\pi - 2\alpha$, where $\alpha = \sin^{-1}\left(\frac{u\sin\theta}{v}\right)$.

Let t_1 hours and t_2 hours be the times taken by the boat to intercept the ship along these two paths. Show that $t_1 + t_2 = \frac{2lu\cos\theta}{u^2 - v^2}$.

12. (a) The trapezium $ABCD$ shown in the figure is a vertical cross-section through the centre of gravity of a smooth uniform block of mass $2m$. The lines AD and BC are parallel, and the line AB is a line of greatest slope of the face containing it. Also, $AB = 2a$ and $B\hat{A}D = \alpha$, where $0 < \alpha < \frac{\pi}{2}$ and $\cos\alpha = \frac{3}{5}$. The block is placed with the face containing AD on a smooth horizontal floor.



A light inextensible string of length l (> $2a$) passes over a small smooth pulley at B, and has a particle P of mass m attached to one end and another particle Q of the same mass m attached to the other end. The system is released from rest with the string taut, the particle P held at the mid-point of AB and the particle Q on BC, as shown in the figure.

Show that the acceleration of the block relative to the floor is $\frac{4}{17}g$ and find the acceleration of P relative to the block.

Also, show that the time taken by the particle P to reach A is $\sqrt{\frac{17a}{5g}}$.

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(*b*) Two particles A and B, each of mass m are attached to the two ends of a light inextensible string of length $l(>2\pi a)$. A particle C of mass $2m$ is attached to the mid-point of the string. The string is placed over a fixed smooth sphere of centre O and radius a with the particle C at the highest point of the sphere, and the particles A and B hanging freely in a vertical plane through O, as shown in the figure. The particle C is given a small displacement on the sphere in the same vertical plane, so that the particle A moves downwards in a straight line path. As long as the particle C is in contact with the sphere, show that $\dot{\theta}^2 = \frac{g}{a}(1-\cos\theta)$, where θ is the angle through which OC has turned. Show further that the particle C leaves the sphere when $\theta = \frac{\pi}{3}$.

13. One end of a light elastic string of natural length a and modulus of elasticity mg is attached to a fixed point O at a height $3a$ above a smooth horizontal floor and the other end is attached to a particle of mass m. The particle is placed near O and projected vertically downwards with speed $\sqrt{ga}$. Show that the length of the string x satisfies the equation $\ddot{x} + \frac{g}{a}(x-2a) = 0$ for $a \le x < 3a$, and find the centre of this simple harmonic motion.

Using the Principle of Conservation of Energy for the downward motion of the particle until the first impact with the floor, show that $\dot{x}^2 = \frac{g}{a}(4ax - x^2)$ for $a \le x < 3a$.

Taking $X = x - 2a$, express the last equation in the form $\dot{X}^2 = \frac{g}{a}\left(A^2 - X^2\right)$ for $-a \le X < a$, where A is the amplitude to be determined.

What is the velocity of the particle just before the first impact with the floor?

The coefficient of restitution between the particle and the floor is $\frac{1}{\sqrt{3}}$. For the upward motion of the particle after the first impact, until the string becomes slack, it is given that $\dot{X}^2 = \frac{g}{a}\left(B^2 - X^2\right)$ for $-a \le X < a$, where B is the amplitude of this new simple harmonic motion to be determined.

Show that the total time during which the particle performs downwards and upwards simple harmonic motions described above is $\frac{5\pi}{6}\sqrt{\frac{a}{g}}$.

14.(*a*) The position vectors of two distinct points A and B with respect to a fixed origin O, **not collinear** with A and B, are $\mathbf{a}$ and $\mathbf{b}$ respectively. Let $\mathbf{c} = (1-\lambda)\,\mathbf{a} + \lambda\,\mathbf{b}$ be the position vector of a point C with respect to O, where $0 < \lambda < 1$.

Express the vectors $\overrightarrow{AC}$ and $\overrightarrow{CB}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and λ.

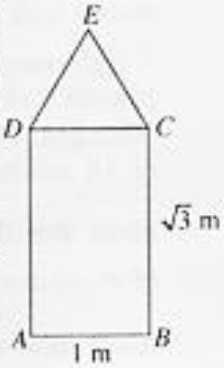
Hence, show that the point C lies on the line segment AB and that $AC : CB = \lambda : (1-\lambda)$.

Now, suppose that the line OC bisects the angle AOB. Show that $|\mathbf{b}|(\mathbf{a}\,.\,\mathbf{c}) = |\mathbf{a}|(\mathbf{b}\,.\,\mathbf{c})$ and **hence**, find λ.

(*b*) In the figure, $ABCD$ is a rectangle with $AB = 1$ m and $BC = \sqrt{3}$ m, and CDE is an equilateral triangle. Forces of magnitude 5, $2\sqrt{3}$, 3, $4\sqrt{3}$, P and Q newtons act along BA, DA, DC, CB, CE and DE respectively, in the directions indicated by the order of the letters. This system of forces reduces to a couple. Show that $P = 4$ and $Q = 8$, and find the moment of this couple.

Now, the directions of forces acting along BA and DA are reversed, but their magnitudes remain the same. Show that the new system reduces to a single resultant force of magnitude $2\sqrt{37}$ newtons.

Show further that the distance from A to the point at which the line of action of this resultant force meets BA produced is $\frac{7}{4}$ m.



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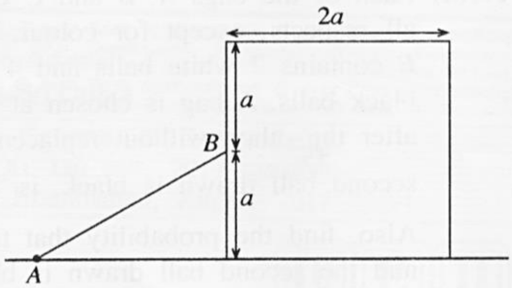
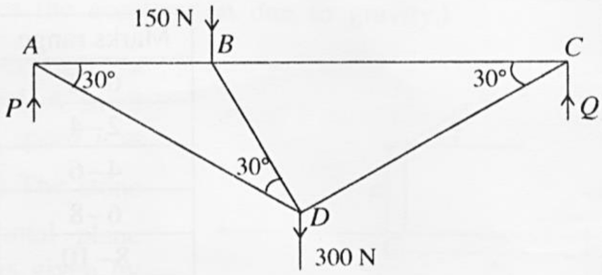
15. (*a*) A uniform cubical block of weight W and side of length $2a$ is placed on a rough horizontal floor. A uniform rod AB of weight $2W$ and length $2a$ has its end A smoothly hinged to a point on the horizontal floor and has the end B against a smooth vertical face of the cube at its centre. The vertical plane through the rod is perpendicular to that vertical face of the block and the system stays in equilibrium. (See the figure for the relevant vertical cross-section.) The coefficient of friction between the cubical block and the rough horizontal floor is μ. Show that $\mu \geq \sqrt{3}$.



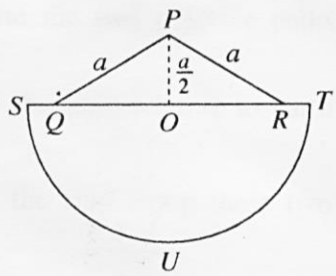
(*b*) The figure shows a framework consisting of five light rods AB, BC, AD, BD and CD freely jointed at their ends. $AB = a$ metres and $BC = 2a$ metres, and $B\hat{A}D = B\hat{D}A = B\hat{C}D = 30°$. The framework is loaded with weights 150 N at B and 300 N at D. It is in equilibrium in a vertical plane supported by two vertical forces P and Q at A and C respectively, so that AB and BC are horizontal. Show that $P = 250$ N.



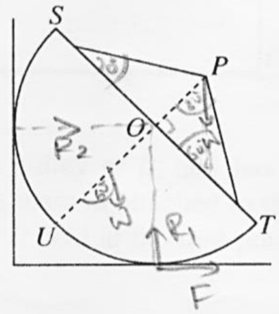
Draw a stress diagram using Bow's notation and **hence**, find the stresses in all the rods and state whether they are tensions or thrusts.

16. Show that the centre of mass of a thin uniform wire in the shape of a semi-circular arc of centre C and radius a, is at a distance $\dfrac{2a}{\pi}$ from C.

In the adjoining figure, PQ, PR and ST are three straight line pieces cut from a thin uniform wire of mass ρ per unit length. The two pieces PQ and PR are welded to each other at the point P and then welded to ST at the points Q and R. It is given that $PQ = PR = a$, $ST = 2a$ and $PO = \dfrac{a}{2}$, where O is the mid-point of both QR and ST. Also, SUT is a semicircular arc of centre O and radius a made up of a thin uniform wire of mass $k\rho$ per unit length, where $k\,(>0)$ is a constant. The rigid plane wire-frame L shown in the figure has been made by welding the semicircular wire SUT to the wire ST in the plane of PQR at the points S and T. Show that the centre of mass of L is at a distance $\left(\dfrac{\pi k + 4k + 3}{\pi k + 4}\right)\dfrac{a}{2}$ from P.



The wire frame L is in equilibrium with its circular part touching a smooth vertical wall and a horizontal ground rough enough to prevent slipping, and its plane perpendicular to the wall as shown in the adjoining figure. Mark the forces acting on L and show that $k > \dfrac{1}{4}$.



Now, let $k = 1$. The equilibrium is maintained in the above position even after a particle of mass m is attached to L at the point P. Show that $m < 3\rho a$.

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17. (*a*) Each of the bags A, B and C contains only white balls and black balls which are identical in all respects, except for colour. The bag A contains 4 white balls and 2 black balls, the bag B contains 2 white balls and 4 black balls, and the bag C contains m white balls and $(m+1)$ black balls. A bag is chosen at random and two balls are drawn from that bag at random, one after the other, without replacement. The probability that the first ball drawn is white and the second ball drawn is black, is $\frac{5}{18}$. Find the value of m.

Also, find the probability that the bag C was chosen, given that the first ball drawn is white and the second ball drawn is black.

(*b*) The following table gives the distribution of marks obtained by a group of 100 students for their answers to a Statistics question:

Marks range	Number of students
0 – 2	15
2 – 4	25
4 – 6	40
6 – 8	15
8 – 10	5

Estimate the mean μ and the standard deviation σ of this distribution.

Also, estimate the coefficient of skewness κ defined by $\kappa = \frac{3(\mu - M)}{\sigma}$, where M is the median of the distribution.

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