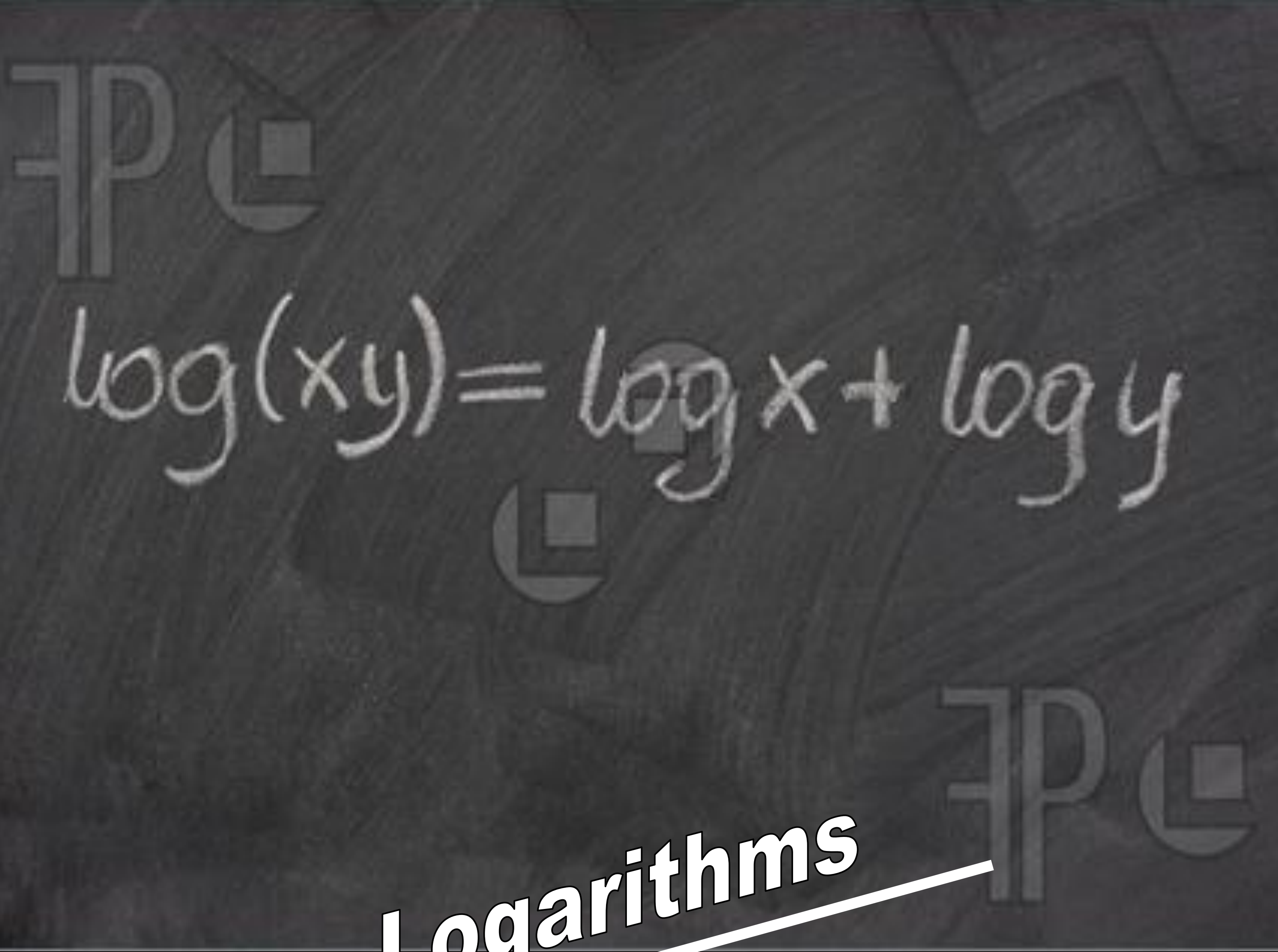
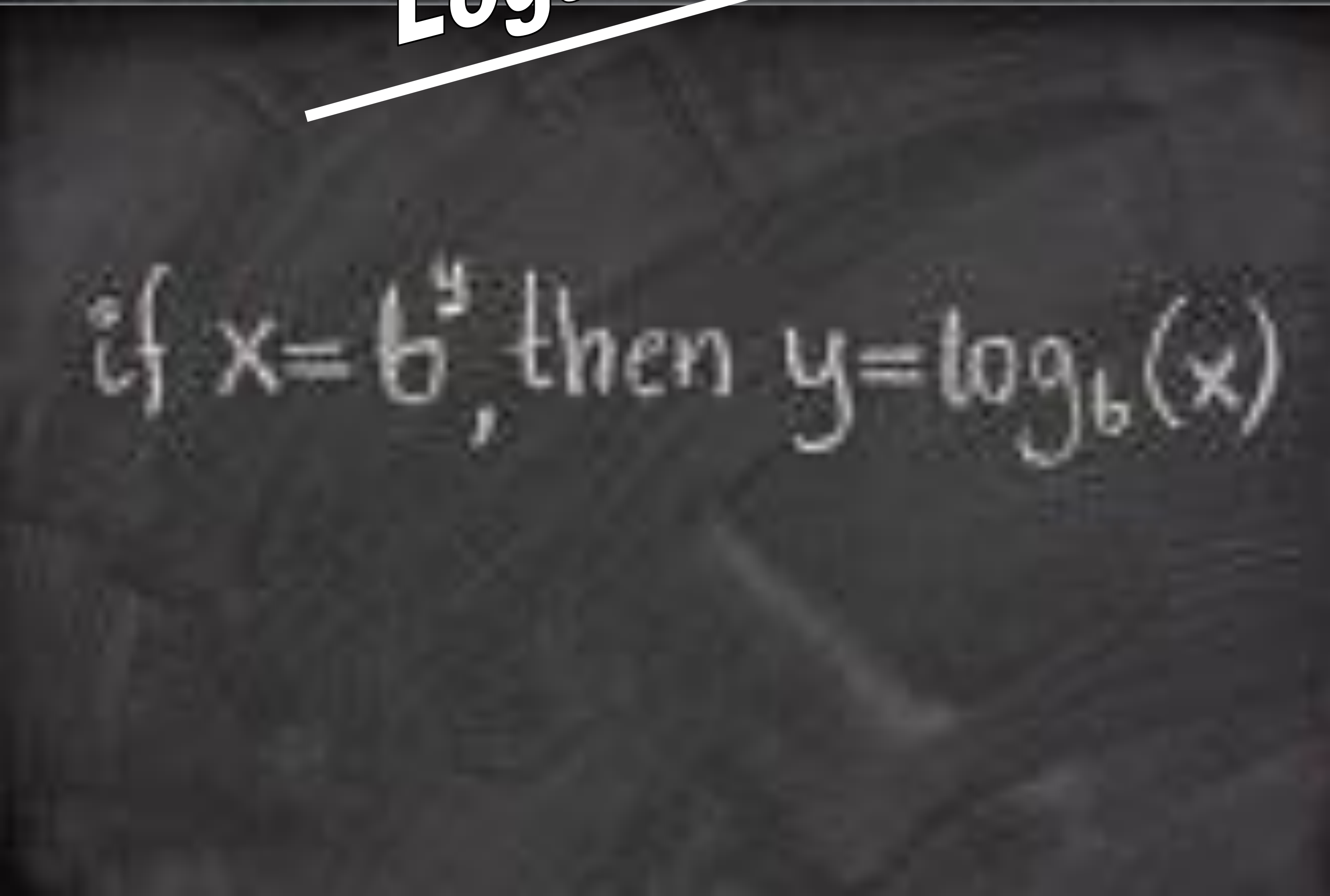


Logarithms



LOGARITHMS

DEFINITION OF LOGARITHMS

$$\text{If } a^x = N \Leftrightarrow x = \log_a N$$

PROPERTIES OF LOGARITHMS

i) $\log_a (mn) = \log_a (m) + \log_a (n)$

ii) $\log_a ((m/n) = \log_a (m) - \log_a (n)$

iii) $\log_a (m^k) = k \log_a (m)$

iv) $\log_a (m) = 1 / \log_m (a)$

v) $\dfrac{\log_c b}{\log_c a} = \log_a (b)$

vi) $\log_a (1) = 0$

vii) $m = a^{\log_a m}$

COMMON LOGARITHMS

Logarithms to the base 10 are known as common Logarithms.

Eg. $10^3 = 1000 \Rightarrow \log_{10} 1000 = \log_{10} 10^3 = 3$

Nermally this is written as log 1000 = 3 without writing 10. Log a means the logarithms to the base 10.

LOGARITHMIC LAWS

If x and y are any two positive values

i) $\log_a (xy) = \log_a x + \log_a y$

ii) $\log_a (x/y) = \log_a x - \log_a y$

iii) If k is any real number $\log_a (x^k) = k \log_a x$

PROOF:

Let $log_a x = p$ and $log_q y = q$

Then $x = a^p$ and $y = a^q$ – by definition

i) $$x \times y = a^p \times a^q = a^{p+q}$$

$$log_a (xy) = p + q \text{ (by definition)}$$

$$\therefore\ log_a (xy) = log_a x + log_a y$$

ii) $$\frac{x}{y} = \frac{a^p}{a^q} = a^{p+q}$$

$$log_a \left(\frac{x}{y}\right) = p - q \text{ (by definition)}$$

$$= log_a x - log_a y$$

CHANGE OF BASE OF LOGARITHMS

1) $$log_a b = \frac{1}{log_b a}$$

PROOF:

If we take $log_a b = c$

by definition $b = a^c$

$\therefore\ a = b^{1/c}$

by definition $log_b a = \frac{1}{c}$.

$$c = \frac{1}{log_b a}.$$

$$log_a b = \frac{1}{log_b a}.$$

$$\therefore \quad \boxed{log_a b = \frac{1}{log_b a}.}$$

2) $$\log_a b = \frac{\log_c b}{\log_c a}$$

PROOF:

If we take $\log_a b = x$

Then it can be written as $b = a^x$

If the logarithm of b to the base c is taken,

$$\log_c b = \log_c(a^x) = x \log_c a$$

$$\therefore \quad x = \frac{\log_c b}{\log_c a}$$

$$\boxed{\log_a b = \frac{\log_c b}{\log_c a}}$$

Deduce $\log_a b = \dfrac{1}{\log_b a}$

$$\log_a b = \frac{\log_c b}{\log_c a}$$

If we take $c = b \Rightarrow \log_a b = \dfrac{\log_b b}{\log_b a} = \dfrac{1}{\log_b a} \Rightarrow \boxed{\log_a b = \dfrac{1}{\log_b a}}$

EXERCISES:

1. ***Prove that***

$$\frac{\log 3\sqrt{3} + \log 2\sqrt{2} - \log 5\sqrt{5}}{\log 1.2} = 3/2$$

2. ***Solve the simultaneous equations given below***

$$i)\ 3(2^x) = 3^y \qquad ii)\ x^2 - y + 1 = 0$$

3. *If* $\log_b a + \log_c a = 2\log_b a \,.\, \log_c a$, ***show that*** $a^2 = bc$

4. ***Prove that*** $4\log_4 6 = 3\log_8 9 + 2$

5. *If* $a = \log(10/9)$, $b = \log(25/24)$, $c = \log(81/80)$, ***then show that*** $\log 2 = 7a - 2b + 3c$

6. *If* $a^2 + b^2 = 7ab$ ***then show that***

$$2\log\{\{(a+b)/3\} = \log a + \log b$$

7. *If* $\log_z x$, $\log_y z$, $\log_x y$ ***lie on an arithmetic progression, where the middle term is a and the common difference is d, show that,*** $ad^2 = a^3 - 1$

8. ***Solve these simultaneous equations.***

$$2^x - 5^y = 3\,,\ 2^{x-3} + 5^{y-2} = 21$$

9. *If* $10^x + 10^{-x} = 4$, ***prove that*** $x = \log_{10}(2 + \sqrt{3})$.

10. Whatever the base is, prove that

$$\log ab = \log a + \log b \ , \ \log (a)^b = b\log a$$

If $p = \log_{10} 2$ *and* $q = \log_{10} 3$ *, find each of the following*

in terms of p and q:

$$\log_{10} 108 \ , \ \log_{10} 10.8 \ , \ \log_{10} \sqrt{(3/5)}$$

11. If a, b, c and d are positive numbers, show that

$$\log (a/b) . \log (c/d) = \log (a/c) .\log (b/d) + \log (a/d) .\log (c/b)$$

12. If a and b are positive integers, express the expression

$\log_{10} 110 + 2 \log_{10} 90 - \log_{10} 55 - 4 \log_{10} 24$, *in the form of* $a - b \log_{10} 2$

13. Solve $7^{2x} - 7^{x+2} + 48 = 0$

14. Prove the following:

$$\frac{\log 27 + \log 8 - \log 125}{\log 6 - \log 5} = 3$$

15. If $x = \log_a (9/8)$, $y = \log_a (16/15)$, $z = \log_a (24/25)$ *then show that*

$$\log_a 2 = 3x + 4y - 2z \ , \ \log_a 3 = 5x + 6y - 3z \ , \ \log_a 5 = 7x + 9y - 5z$$

16. If $n \neq 1$ *such that* $\log_a n = x$ *and* $\log_c n = y$*, show that*

$$\frac{x - y}{x + y} = \frac{\log_b c - \log_b a}{\log_b c + \log_b a}$$

17. *Solve* $\log_3 x - 2\log_x 3 = 1$

18. *when a and b are inequal positive numbers in*
$\log_a b + \log_b a^2 = 3$, *find b in terms of a.*

$b = a^x$

19. *If* $x = \log_a b$, *show that* $\log_s t = \log_r t / \log_r s$
If $\log_3 6 = m$ *and* $\log_6 5 = n$, *find the value of* $\log_3 10$,
in
terms of m and n.

20. *Solve the simultaneous equations* $\log_y x = 2$, $5y = x + 12 \log_x y$

21. *Solve* $\log_2 x + \log_2 y = 3$, $\log_y x = 2$

22. *Solve these simultaneous equations*
$\log (x-2) + \log 2 = 2 \log y$, $\log (x - 3y + 3) = 0$

23. *Show that* $\log_{16} (xy) = 1/2 \log_4 x + 1/2 \log_4 y$. *Hence, or otherwise solve* $\log_{16} (xy) = 3\ 1/2$, $\log_4 x / \log_4 y = 8$

24. *Solve the simultaneous equations* $\log_9 (xy) = 5/2$,
$\log_3 x \log_3 y = -6$

25. Solve the following:

(a) $9 \log_x 5 = \log_5 x$

(b) $\log_8 x/2 = \log_8 x / \log_8 2$

26. (i) Show that $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \dots\dots\dots + \frac{1}{\log_{43} n} = \frac{1}{\log_{(43!)} n}$.

(ii) Show that if $\frac{\log a}{(b-c)} = \frac{\log b}{(c-a)} = \frac{\log c}{(a-b)}$ then $a^a b^b c^c = 1$

(iii) Show that $\frac{1}{[(\log_a bc) + 1]} + \frac{1}{[(\log_b ca)+1)]} + \frac{1}{[(\log_c ab)+1]} = 1$

27. If, a,b and c are distinct positive numbers each different from 1 such that

$(\log_b a \log_c a - \log_a a) + (\log_a b \log_c b - \log_b b) + (\log_a c \log_b c - \log_c c) = 0$

then prove that abc = 1.

28. $\log_{10}[98 + \sqrt{(x^2 - 12x + 36)}] = 2$

29. Show that $\log_{10}(1 + 2 + 3) = \log_{10} 1 + \log_{10} 2 + \log_{10} 3$

30. Show that $x^{(\log y - \log z)} \cdot y^{(\log z - \log x)} \cdot z^{(\log x - \log y)} = 1$

31. Show that $7 \log (16/15) + 5 \log (25/24) + 3 \log (81/80) = \log 2$

32. Solve $\log_3 x + \log_9 x^2 + \log_{27} x^3 = 3$

33. Show that $(\log_4 32) \cdot (\log_{16} \quad) \cdot (\log_{64} 2) = 5/24$

34. Show that

$$\frac{1}{\log_2 2001} + \frac{1}{\log_3 2001} + \frac{1}{\log_4 2001} + \dots\dots + \frac{1}{\log_{100} 2001} = \frac{1}{\log_{(100!)} 2001}.$$

35. If $\log_2 [\log_2 (\log_2 x)] = 1$, find x.

36. Find the value of $\log_2 [\log_2 \{ \log_3 (\log_3 27^3) \}]$

37. Solve $2 \log_x a + \log_{ax} a + 3 \log_{a^2 x} a = 0$

38. If $x = \log_{2a} (a)$, $y = \log_{3a} (2a)$, $z = \log_{4a} (3a)$, then prove that $xyz + 1 = 2yz$.